P P SAVANI UNIVERSITY

Third Semester of B. Tech. Examination
May 2019
SESH2011 Differential Equations

16.05.2019, Thursday **Time: 09:00 a.m. To 11:30 a.m.** **Maximum Marks: 60**

Instructions:

1. The question paper comprises of two sections.
2. Section I and II must be attempted in separate answer sheets.
3. Make suitable assumptions and draw neat figures wherever required.
4. Use of scientific calculator is allowed.

SECTION – I

Q - 1 Answer the following. (Any Five) **[05]**

(i) D'Alembert's solution of the wave equation is,

a) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ b) $\frac{\partial^2 u}{\partial t^2} = c \frac{\partial^2 u}{\partial x^2}$ c) $\frac{\partial^2 u}{\partial t^2} = 2c \frac{\partial^2 u}{\partial x^2}$ d) None of these

(ii) Solution of one-dimensional heat flow equation is

a) $u = (c_1 \cos mx + c_2 \sin nx)e^{n^2m^2c^2t}$ b) $u = (c_1 \cos mx + c_2 \sin mx)e^{m^2c^2t}$

c) $u = (c_1 \cot mx + c_2 \sin nx)e^{n^2m^2c^2t}$ d) None of these

(iii) The order & degree of the differential equation $\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$ are respectively

a) $\frac{3}{2}, 2$ b) 2, 3 c) 2, 2 d) 3,4

(iv) An integrating factor of differential equation $(x^2y - 2xy^2)dx = (x^3 - 3x^2y)dy$ is

a) $\frac{1}{xy}$ b) xy c) x^2y^2 d) $\frac{1}{x^2y^2}$

(v) For the differential equation in the form of $Pp + Qq = R$ subsidiary equation is

a) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ b) $\frac{dx}{P} = -\frac{dy}{Q} = \frac{dz}{R}$ c) $\frac{dx}{P^2} = \frac{dy}{Q^2} = \frac{dz}{R^2}$ d) $\frac{dx}{P^2} = -\frac{dy}{Q^2} = \frac{dz}{R^2}$

(vi) Solution of the PDE: $z = px + qy + \sqrt{pq}$

a) $z = ax + by + \sqrt{ab}$ b) $z = ax + by + ab$

c) $z = ax + by$ d) None of these

Q - 2 (a) Answer the following. (Any Two) **[04]**

1. Solve PDE: $\frac{\partial^2 z}{\partial x \partial y} = \frac{x}{y} + a$.
2. Solve PDE: $yzp - xzq = xy$.
3. Form the differential equation by eliminating arbitrary constants from $\log\left(\frac{y}{x}\right) = cx$.

Q - 2 (b) Answer the following. (Any Two) **[06]**

1. Define General solution of ODE & solve IVP: $y' = -2xy, y(0) = 1.8$
2. Solve PDE: $yzp - xzq = xy$.
3. Solve ODE: $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 12y = e^{6x}$.

Q – 3 Answer the following. (Any Three) **[05]**

(i) Solve ODE: $(2x - 2y + 5)dy - (x - y + 3)dx = 0$.

(ii) Find the general solution of $y'' + y = 32x^3$ using method of undetermined coefficients.

(iii) Solve PDE: $(D^3 - 3D^2D' + 4D'^3)z = e^{x+2y}$.

(iv) Find orthogonal trajectories of the family of confocal conics $\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda}$, where λ is parameter.

SECTION – II

Q - 1 Answer the following. (Any Five) **[05]**

(i) $L[te^{-t}]$ is

a) $\frac{1}{s^2}$ b) $\frac{1}{(s+1)^2}$ c) $\frac{2}{(s+1)^3}$ d) None of these

(ii) Laplace transform of unit impulse function $\delta(t-a)$ is

a) e^{-as} b) 1 c) e^{-s} d) 0

(iii) The value of b_n in the Fourier series expansion $f(x) = x^2$ in $(-\pi, \pi)$ is

a) 0 b) 2π c) $\frac{\pi}{2}$ d) π

(iv) The value of a_n in the Fourier series of $f(x) = x - x^3$ in $(-\pi, \pi)$ is

a) $\frac{\pi}{4}$ b) 2π c) $\frac{\pi}{2}$ d) 0

(v) Which of the following is an even function of t?

a) t^2 b) $t^2 - 4t$ c) $\sin 2t - 4t$ d) $t^3 + 6$

(vi) A "periodic function" is given by a function which

a) has a period $T = 2\pi$ c) satisfies $f(t+T) = f(t)$

b) satisfies $f(t+T) = f(t)$ d) has a period $T = -2\pi$

Q - 2 (a) Answer the following. (Any Two) **[04]**

1. If $L\{f(t)\} = \frac{8(s-3)}{(s^2-6s+25)^2}$, find $L\{f(2t)\}$.
2. Find $L\left(\frac{e^{at}-1}{a}\right)$ & $L(\sin 2t \cos 3t)$.
3. Define Fourier Sine & Cosine Integral.

Q - 2 (b) Answer the following. (Any Two) **[06]**

1. Write Fourier series expansion of even & odd function.
2. Find the value of a_0 & a_n for $f(x) = x^{11}$, $where\ -\pi \le x \le \pi$
3. Using method of partial fraction calculate $L^{-1}\left(\frac{6}{(s+2)(s-4)}\right)$

Q - 3 Answer the following. (Any Three) **[05]**

(i) Find the Fourier series expansion of the function $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$.

(ii) Find the Laplace transform of $f(t) = \begin{cases} 0; & 0 \le t < 1 \\ t; & 1 \le t < 4 \\ 0; & t \ge 4 \end{cases}$.

(iii) Solve: $2\frac{dx}{dt} + \frac{dy}{dt} - x - y = e^{-t}, \frac{dx}{dt} + \frac{dy}{dt} + 2x + y = e^t, x(0) = 2, y(0) = 1.$

(iv) Find the Fourier sine transform of f(t), where $f(t) = \begin{cases} t, & 0 \le t \le 1 \\ 0, & t > 1 \end{cases}$

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